

OKLAHOMA STATE UNIVERSITY  
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING



ECEN 4413  
Automatic Control Systems  
Spring 2009



Midterm Exam #2

Choose any four out of five problems.  
*Please specify which four listed below to be graded*

1) \_\_\_\_\_; 2) \_\_\_\_\_; 3) \_\_\_\_\_; 4) \_\_\_\_\_;  
:

Name : \_\_\_\_\_

E-Mail Address: \_\_\_\_\_

**Problem 1:**

The equations that describe the dynamics of a motor control system are

$$e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + K_b \frac{d\theta_m(t)}{dt}$$

$$T_m(t) = K_t i_a(t)$$

$$T_m(t) = J \frac{d^2\theta_m(t)}{dt^2} + B \frac{d\theta_m(t)}{dt} + K\theta_m(t)$$

$$e_a(t) = K_a e(t)$$

$$e(t) = K_s [\theta_r(t) - \theta_m(t)]$$

- a) Assign the state variables as  $x_1(t) = \theta_m(t)$ ,  $x_2(t) = d\theta_m(t)/dt$ , and  $x_3(t) = i_a(t)$ .

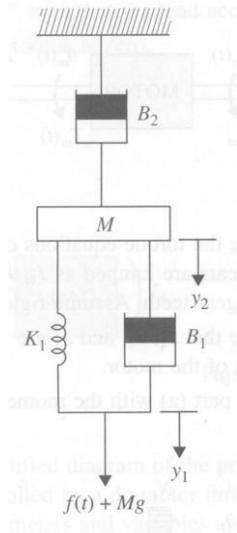
Express the state space representation in the form of

$$\frac{dx(t)}{dt} = Ax(t) + B\theta_r(t), \quad \theta_m(t) = Cx(t).$$

- b) Develop a corresponding state diagram.
- c) Find the transfer function  $G(s) = \Theta_m(s)/E(s)$  when the feedback path from  $\Theta_m(s)$  to  $E(s)$  is broken. Find the closed-loop transfer function,  $M(s) = \Theta_m(s)/\Theta_r(s)$ .

**Problem 2:**

Write the equation of motion for the linear translational system shown below. Draw the state diagram using a minimum number of integrators. Write the state equation from the state diagram. Find the transfer functions  $Y_1(s)/F(s)$  and  $Y_2(s)/F(s)$ . Set  $Mg = 0$  for the transfer function.



**Problem 3:**

For the system described by input-output differential equation given below,

$$c\ddot{y} = (a+1)y + \dot{y} - b\ddot{u} + 2u + \dot{u} + \ddot{u},$$

find the state space representation in the form of

$$\dot{x}(t) = Ax(t) + bu(t)$$

$$y(t) = cx(t) + du(t)'$$

where input is  $u(t)$  and output is  $y(t)$ .

**Problem 4:**

For the state variable description,

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$$

if  $u(t) = e^{-3t} u_s(t)$ , where  $u_s(t)$  is the unit step function and initial conditions are all zeros, find  $y(t)$ .

**Problem 5:**

Derive the state space representation of the system in the state diagram shown below

$$\dot{x}(t) = Ax(t) + Br(t), \quad y(t) = Cx(t).$$

Please note the initial conditions are ignored here.

